A Comparative Study between ARDL and Johansen Procedures in Narrow Money Estimation in the Iranian Economy

Mahdi Mostafavi (Ph.D.) *

Received: 2011/7/9                Accepted: 2012/3/8

Abstract: The purpose of this study is estimation of narrow money in the long run via ARDL and Johansen procedures. In doing so first econometric model has been investigated. For modeling narrow money two studies have been applied namely Rother (1999) and Bahmani Oskooee (1996). The applied model in this study deals with GDP, inflation, rates of return on foreign exchange and cars. In order to investigate the long run relationship among involved determinants in the model, cointegration test has been applied via ARDL and Johansen procedures. According to λ trace adjusted there is only one cointegration vector for the RMI model. Then we derived the long run coefficients for RMI model and concluded the same results as those of the ARDL test in the light of the theory basis. Although the ARDL and Johansen use very different techniques in estimation, the former employs ordinary least squares while the latter uses the maximum likelihood estimation method. Thus, one-to-one comparison of the magnitudes of the coefficients may not be appropriate and requires some caution. Hence there is a bit difference between two mentioned results in determinants parameters magnitudes. Explanation of the coefficients of determinants (GDP and inflation) in Johansen is more real and closer to theory in comparison to ARDL procedure.

JEL classification: C52

Keywords: Real narrow money, long run, cointegration, ARDL, Johansen

* Faculty member of economics at Ferdowsi University of Mashhad, Mashhad, Iran. Email: (mostafavimahdi@yahoo.co.uk)
1. Introduction
The paper’s objective is to provide a comparative discussion for two different cointegration procedures ARDL and Johansen for estimating real narrow money (RM1) in the Iranian economy.

Demand for narrow money has attracted the attention of monetary economists since the mid-eighteenth century or even earlier, because it relates to the main macroeconomic issues of inflation, unemployment, levels of income, interest rates, financial markets and the banking system. Basically, demand for money theories can be analysed under two major headings:

A. The transactions demand for money: this type of demand for narrow money happens when there is a distance of time between income and expenditures. The transaction motive of demand for money can usually be explained via real wealth, permanent income or current income (depending on the theory).

B. Opportunity cost of holding money: this type of demand occurs when an individual makes a comparison between the rate of return on narrow money and its proxies (bonds, equities, and durable goods). The opportunity cost of holding money is related to the role of money as a store of value. The rate of interest, the rate of inflation or the rate of exchange in the price of durable goods, are relevant factors here.

According to Friedman (1956) and Goldfeld (1973), the theoretical specification for the real narrow money function is given by:

\[ m = f(i, \frac{Y}{P}) \]  

(1)

where \( m \), \( Y \), \( i \) and \( P \) stand respectively for real money stock, income, nominal interest rate and price level. Hence a numerous authors\(^1\) believe that in developing countries the possible effective determinants on MR1 are GDP and inflation.

---


Since time series data are normally accomplished with the issue of non stationary problem, we should pay attention to the issue of the stationarity. Time series authors believe that when two or more series are integrated of the same order, they are cointegrated, and there could be a long run relationship among them. Hence econometricians supply some procedures in order to determine the magnitude effect of appropriate factors on the dependent variable. Hence this study intends to discuss two major procedures, namely ARDL and Johansen to estimate cointegration among RM1 and its determinants.

This paper is organized in 8 sections. Section 1 is introduction. Section two devotes to theoretical debates of the topic. Section 3 discusses the methodology of the research and database. Section 4 reviews a short report of the existing studies for the topic throughout the world. Section 5 carries out the data analysis. Section 6 includes preferred model in this study. Section 7 tries to do empirical studies via ARDL and Johansen procedure, and finally section 8 is devoted to concluding remarks.

2. Theoretical Basis

To define cointegration, one simply can say if the nonstationary \( y_t \) and \( x_t \) sequences are integrated of the same order and the error term is stationary, \( y_t \) and \( x_t \) are cointegrated (Enders, 1995: 219).

If \( y_t \) and \( x_t \) are cointegrated, the OLS estimation of this equation might possibly supply a super-consistent estimate of the parameters involved in the equation (Thomas, 1997: 428). The residuals of the equation constructed by \( y_t \) and \( x_t \) should then be calculated. If the residuals become stationary, there is a long run relationship among the mentioned cointegrated series. The long run relationship among variables in an econometric model has been investigated via several ways. Some those conventional ways are Engle – Granger, ARDL, and Johansen procedures.

Although the Engle and Granger (1987) procedure is easily accessible, it has some important defects. First, the researchers using this procedure face the question of how they should chose one variable for the left-hand side as a dependent variable,
leaving the other variables on the right hand side as explanatory variables. This defect occurs when the economic theory does not tell us which variable is the dependent one. As Hafer and Jansen (1991) and Kennedy (1992) state, choosing a wrong dependent variable will usually affect the estimation results. The second serious defect in this procedure is the two-step estimator. In the first step, we suppose that the variables are cointegrated and, by accepting this assumption, we obtain the residuals. In the second step, the residual sequence is examined for stationarity. But we cannot postulate that $x_t$ and $y_t$ are cointegrated, unless we prove it before. Doing things in this sequence is not logical. As Dickey et al. (1995) state, using this procedure it is difficult to reject the null hypothesis of non-stationarity for error terms.

The third defect of the Engle and Granger (1987) procedure is lack of knowledge of the correct number of long-run relationships among variables (Hafer and Jansen, 1991). This problem emerges when there are more than two variables in the model, because, as Miller (1991) points out, the cointegration vector of two-variable equations is unique, while it is not necessarily unique when there are several variables in the model.

As a result emphasize should be put on two last procedures. ARDL solves the second defect of Engle - Granger and Johansen solves all three defects of Engle - Granger. The next stage we will explain the methodology of the procedures.

3. Methodology and database

In order to estimate the dependent variable, two different procedures namely ARDL and Johansen could be applied. As Charemza and Deadman (1992) state, the unrestricted ARDL model for the two variables $y_t$ and $x_t$ is as follows:

$$y_t = \sum_{i=1}^{n} a_i y_{t-i} + \sum_{j=0}^{n} \beta_j x_{t-j} + e_t$$ (2)

where the cointegration coefficient is obtained by the formula:
\[ \beta^* = \frac{\sum_{j=0}^{n} \beta_j}{1-\sum_{i=1}^{n} a_i} \]  \hspace{1cm} (3)

Now \( \beta^* \) can take the place of \( \beta \) in the ECM. This is a briefly explanation of ARDL procedure. Now one should go to explain Johansen procedure. As Harris (1995) states, the estimation of a series like RM1 in the long run will be carried out via three stages as follows:

i. Estimating and evaluating vector auto-regression (VAR) models

ii. Testing for cointegration, (i.e. testing for the number of long run relationships among variables)

iii. Testing restrictions on \( \beta \) matrix

Looking first at the available data, the data for Iran is usually presented in an annual publication called Iran Statistical Yearbook, prepared by the Statistical Centre of Iran. As for the accuracy of the data, given that the Central Bank of Iran is the oldest and most accurate data source, most of the necessary data for this study (such as M1, M2, and the price of durable goods and the CPI) and other national accounts data are taken either from the Central Bank’s bulletins, or via its website at www.cbi.ir.

4. A Review of the literature

Obviously a plenty of investigations have carried out for estimation of RM1 via the mentioned procedures throughout the world.

Metin (1994) estimated the real narrow money in the Turkish economy using quarterly data for the period 1948:1 to 1987:4. The set of explanatory variables includes GNP prices, the rate of inflation and the Central Bank nominal discount rate and the functional form being double log.

Metin (1994) applied time series and cointegration in his study. A unit root ADF test showed that narrow money, GNP, inflation and interest rate are integrated of order one, I(1).
applying the Johansen (1988) maximum likelihood method, it was found that in the long run, the income and inflation elasticities are 2.8 and –0.98 respectively. There is no statistically significant long run effect, although the interest rate coefficient in the short run demand for money was significant during the whole period. Moreover there is a high gap between income elasticity values in the long run and short run (2.8 and 0.44). This implies that the speed of adjustment should be very slow.

Choudhry (1995) estimated the demand for narrow money in Argentina using quarterly data for the period 1935 to 1962 and 1946 to 1962. These periods coincided with important economic transitions and with high and volatile inflation. His set of inflationary variables includes income and inflation. Choudhry used time series and co integration in his work. The augmented Dickey Fuller test showed that all of the series are I(1). Then he performed a co integration test. For the longer period eight lags were significant and for the shorter period four lags. Choudhry then formed two econometric models, (M1 and M2). He did empirical work on each of them over both the short term and the long term period. He concluded that there is one co integrating vector between real narrow and broad money, real income, and the rate of inflation for each of the four cases. In all four relationships, the rate of inflation was significantly different from zero at the 1 percent level. Real income is significant only during the long period in both real M1 and real M2 functions. During both periods inflation rates seem to have more effect on the demand for real M2 than on real M1. Real income failed to be significant during the short period.

Bahmani-Oskooee (1996) used a model to estimate RM1. His study has at least two advantages over the most of the others. First, it chooses a black market exchange rate, whereas most researchers either did not apply this factor or employed the official rate along with it. In fact, the actual rate for hard currency effective in RM1 is its rate in the free-market, as the dollar is available to everybody in this market. The second advantage of this study is that it pays attention to the question of stationarity.
Since the factors involved in the demand for money model, (which are on the levels), are not usually stationary, and the differences on variables should be applied in the model. Bahmani-Oskooee (1996) suggested RM1 model as follows:

\[ RM1_t = a + bY_t + cInf_t + dEX_t + e_t \]  \hspace{1cm} (4)

where \( M \) is the demand for real cash balances, \( Y \) is the real GDP, \( Inf \) is inflation, and \( EX \) is the exchange rate defined as the number of Iranian Rials per US dollar.

In this model, the order of integration for the entire involved factors should be determined. The common method for determining the order of integration is the ADF test. Bahmani-Oskooee (1996) argues however that graphs for the factors involved in his model showed the presence of a structural break, (due to the revolution), in each series around 1978, except in the case of the official exchange rate, which is almost constant during most of the period. Hence he has applied the ADF test for the official exchange rate, (OEX), and has also applied a modified version by Perron (1989) who incorporated a structural break for other factors in the model. The results showed that all variables involved in the model are I(1). In order to determine the long run relationship among variables, Bahmani-Oskooee has then applied the Johansen and Juselius (1990) procedure. He has examined it, using two likelihood ratio tests. The results of these tests for different groups of factors showed that when OEX was included in the RM1 model, there was one cointegrating vector, but when the black market exchange rate (BEX) is included, there were two cointegrating vectors in the RM1 equation.

Tavakkoli (1996) estimates the narrow real money function using quarterly data from 1972:1-1990:1. The set of explanatory variables includes real GDP and the inflation rate. Since quarterly data for GDP was not available, Tavakkoli converts annual data to quarterly through Lisman and Sandee’s method. Unit root tests indicated that narrow money and real GDP are integrated at I(1), and that the inflation rate is stationary. The results of Johansen
Cointegration analysis showed that only one cointegrating vector between the real money stock and their determinant exists. The long run GDP elasticity with respect to narrow money is 0.106. The estimated coefficient for the inflation rate is -5.67. Although these two figures have the correct signs, the former has a low value while the latter is high.

There are three problems with Tavakkoli’s work relating to data conversion from annual to quarterly, to cointegration regression and to data analysis. To deal with the first problem, Tavakkoli has applied the procedure of Lisman and Sandee (1964). Yet as Bruggeman (1995) argues, this procedure is highly arbitrary. He has not used GDP components information, while quarterly data for GDP components like oil, agriculture, industry and services are available.

Sharifi-Renani (2008) estimates RM1 using quarterly data for the years 1983 - 2005. The set of explanatory variables in this work includes real income, exchange rate and inflation rate. First he has tested the stability of the model via CUSUM and CUSUMSQ tests. Then he has tested the stationarity of the variables and concluded the all of them are I(1) via Dickey-Fuller test. Then he has estimated the MR1 in the long run via ARDL. He concluded that the elasticities for the real income, exchange rate and inflation rate 2.65, 0.67 and -0.05 respectively.

In justification of the explanatory variable signs he state that the positive sign for real income is due to its substitution for fiscal assets. He also believes that the positive sign of the BEX coefficient could be justified by the wealth effect in the literature. The major defect of this study is its use of the exchange rate rather than the rate of return of foreign exchange.

5. Date analysis
As mentioned before, the order of integration for all series involved in this study should first be determined. Those series are RM1, GDP, rate of return on cars and hard currency and

---

3 see Tavakkoli,1996:124.
inflation. Hence unit root test should be done. The most conventional test is Dickey - Fuller (DF) test.

In the DF model \( x_t = \varphi x_{t-1} + \xi_t \) the \( \varphi = 1 \) has been tested. Subtracting \( x_{t-1} \) from the two sides of this equation gives:

\[
x_t - x_{t-1} = (\varphi - 1) x_{t-1} + \xi_t \tag{5}
\]

This equation is equivalent to:

\[
\Delta x_t = \gamma x_{t-1} + \xi_t \tag{6}
\]

when \( \varphi - 1 = \gamma \). Dickey and Fuller (1979) consider two other regression equations, which can be applied in testing for a unit root. These equations are:

\[
\Delta X_t = a_0 + \gamma X_{t-1} + \xi_t \tag{7}
\]

\[
\Delta X_t = a_0 + \gamma X_{t-1} + a_2 t + \xi_t \tag{8}
\]

The differences in these equations are related to the deterministic components. The first equation (6) is a random walk if the null hypothesis (\( \gamma = 0 \)) cannot be rejected. The second equation (7) is a random walk with drift or a random walk with an intercept. The third equation includes both a drift and linear time trends.

In some cases, there may be an auto-correlation among the error terms. In this circumstance, the OLS method does not provide efficient estimated values for the parameters in the model. One solution is to include some lags of dependent variables in the model. The usual specification of this model is:

\[
\Delta x_t = \gamma x_{t-1} + \sum_{i=1}^{k} \gamma_i \Delta x_{t-i} + \xi_t \tag{9}
\]

We should include so many lags as to make the error term become white noise. Doornik and Hendry (1994a) have provided a procedure to find the appropriate number of lags and contemporaneously the existence of the unit root. [See Charmeza and Deadman (1992:135) and Banerjee et al. (1994:107)]. As a first step, a fairly large number of lags was selected (\( k = 20 \) to 30 is chosen in this study) and then they were dropped one by one, until the null hypothesis for the parameter of lagged variables could not be rejected. As the result the optimum number of lags for Rex and Rcar are 10 and zero respectively. Also the
correspondent number of lags for DRM1, DGDP, and DD4Inf are 3 and 7 respectively.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Calculated Value</th>
<th>Significance</th>
<th>Constant</th>
<th>Trend</th>
<th>Lags</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>95%</td>
<td>99%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rcar</td>
<td>-8.47</td>
<td>-2.91</td>
<td>-3.56</td>
<td>*</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>Rex</td>
<td>-6.33</td>
<td>-2.90</td>
<td>-3.54</td>
<td>*</td>
<td>-</td>
<td>10</td>
</tr>
<tr>
<td>GDP</td>
<td>-1.79</td>
<td>-3.48</td>
<td>-4.12</td>
<td>*</td>
<td>*</td>
<td>4</td>
</tr>
<tr>
<td>GDP</td>
<td>-0.88</td>
<td>-2.91</td>
<td>-3.54</td>
<td>*</td>
<td>-</td>
<td>4</td>
</tr>
<tr>
<td>DGDP</td>
<td>-4.01</td>
<td>-2.91</td>
<td>-3.54</td>
<td>*</td>
<td>-</td>
<td>3</td>
</tr>
<tr>
<td>Inf</td>
<td>-2.33</td>
<td>-2.91</td>
<td>-3.57</td>
<td>*</td>
<td>-</td>
<td>4</td>
</tr>
<tr>
<td>D4Inf</td>
<td>-2.83</td>
<td>-2.91</td>
<td>-3.57</td>
<td>*</td>
<td>-</td>
<td>10</td>
</tr>
<tr>
<td>DD4Inf</td>
<td>-4.19</td>
<td>-2.91</td>
<td>-3.57</td>
<td>*</td>
<td>-</td>
<td>7</td>
</tr>
<tr>
<td>RM1</td>
<td>-2.82</td>
<td>-3.48</td>
<td>-4.12</td>
<td>*</td>
<td>*</td>
<td>4</td>
</tr>
<tr>
<td>DRM1</td>
<td>-7.34</td>
<td>-2.91</td>
<td>-3.54</td>
<td>*</td>
<td>-</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1 shows the corresponding t-statistic values for 1 and 5 percent levels. The number of lags for the dependent variable is in the seventh column. This table includes deterministic components. Real money, GDP and inflation are I(1), but the rates of return on cars and hard currency are stationary.

6. The Preferred Model
The model of this study is driven from the studies of Bahmani-Oskooee (1996) Rother (1999). Three explanatory variables are taken following the model of Bahmani-Oskooee (1996). Another one was taken following Rother (1999). She used the changes in the price of gold as an index for inflation in the countries of West Africa, while for the same purpose I uses changes in car prices instead of gold prices. Throughout the period 1992-2008 the path of inflation was upward, and Iranians speculated in durable goods such as new or used cars. As Ghatak (1995:25) has shown, the wealth holders in developing countries tend either to hold money or real physical assets like buildings and durable goods. Hence the rate of return on cars can help to explain the changes in real
money and is to be included in the model. Thus the model selected for this study has one equation of RM1 based on monetarists viewpoints such as Friedman (1956) and Goldfeld (1973). The determinants in this equation are real GDP, inflation, the rate of return on foreign exchange, and the rate of return on cars. The first determinant shows transaction demand for M1 and the rest ones show opportunity cost for M1.

By paying attention to the previous empirical works in section 4 findings and also theoretical discussions, the money demand function is expressed as follows:

\[
\left( \frac{M_1}{P} \right)_t = a_0 + a_1 RGDP_t + a_2 Inf_t + a_3 REx_t + a_4 RCar_t + u_t
\]  

Where \(M_1/P\) stands for real narrow money, RGDP is real gross domestic product, Inf stands for inflation, REx and RCar stand for rate of return on foreign exchange and rate of return on cars respectively. This equation is used to explain the RM1 process in Iran. Money stock and GDP are in logarithms, but the remaining variables are not, since negative figures have no logarithm, and because \(\ln (1+\pi/100)\) is approximately equal to \(\pi/100\).

7. Empirical Studies

In order to estimate RM1 via ARDL procedure, micofit4 was used.

According to both the Akaike Information criterion and the Schwarz Bayesian criterion the maximum number of lags in RM1 equation is four. The results of the ARDL tests are shown in the following equations.

\[
\hat{RM1} = -0.04 t + 0.38 GDP - 0.19 Inf
\]

As these estimations show the sign of GDP in the model is positive and consistent with theory. The inflation coefficient is negative, meaning that in inflationary conditions, people do not wish to hold money because the purchasing power of money is declining.

---

4 See: Thomas, 1997:456
A useful way to illustrate the Johansen methodology is to implement it by the following three steps.

7.1. To estimate the VAR model
In order to test the existence of cointegration among the variables, various VAR models of the following type are estimated:

$$X_t = \mu + \alpha_t + A_1 X_{t-1} + \ldots + A_k X_{t-k} + \varphi_0 z_t + \varphi_1 z_{t-1} + \ldots + \varphi_k z_{t-k} + u_t$$

$$t = 1, \ldots, T$$

(12)

where $k$ is the number of lags, $X$ is a vector comprising I(1) variables which contain real narrow money (RM1), GDP and inflation (Inf), $Z$ is a vector comprising I(0) variables which are the rates of return on foreign exchange and cars, $\mu$ is a constant vector, and $t$ is a trend vector.

This model can be written as a vector of an error correction model:

$$\Delta X_t = \mu + \varphi_t + \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \ldots + \Gamma_{k-1} \Delta X_{t-k+1} + \pi X_{t-k+1} + \varphi_0 z_t + \varphi_1 z_{t-1} + \ldots + \varphi_k z_{t-k} + U_t$$

(13)

There are several points to consider before estimating the model. The first point concerns the selection of appropriate deterministic components for the VAR model. Equation (13) is applied, but with an unrestricted constant and a trend imposed onto the cointegration space. Thus in accordance to the fourth model suggested by Harris (1995), this is preferred because the first difference series do not have a linear mean. The second point is that since rates of return on foreign exchange and cars are stationary, they are regarded as non-modeled and do not enter in the cointegration space (Banerjee et al., 1994; Thomas, 1997). For this reason we used these variables and their lags in the model as unrestricted. The third point is the differing order of integration in these series. RM1, GDP, and inflation are I(1), while the rates of return on foreign exchange and cars are

---

5 This term means that they cannot be considered as dependent variable.
stationary. According to Enders (1995: 396), forms of the Johansen test can detect differing orders of integration. As shown in the econometric model, we have included nonstationary series in the “X” vector and stationary series in the “Z” vector. The fourth point concerns the appropriate number of lags in this model. As Table 2 indicates, four lags must be added in the model to remove the error terms autocorrelation, since even with three lags the autocorrelation problem occurs (Fau (36, 77) = 2.69**). Moreover, this table indicates a significant four lagged values for real narrow money and inflation. Therefore, four lags of all variables entered into the model, owing to the necessity of an equal lag among all cointegration relationships (Harris, 1995: 82). As a result we formed a VAR model on the basis of these points, and the diagnostics are reported in Table 2. The single equation diagnostic in this table indicates that there is no error autocorrelation (Fau, from lag 1 to 4); that there is no autoregressive conditional heteroscedasticity (Farch, from lag 1 to 4); and that there is no heteroscedasticity problem.

In addition multivariate tests in Table 2 indicate that the VAR model has no autocorrelation, and also no heteroscedasticity. Finally, Fun indicates the significance of fundamental determinants involved in the model (Fun (39, 95) = 144.88**). The only diagnostic test that fails is the normality test for GDP and for the system, and the normality assumption is not so serious for conclusion (Johansen, 1995: 29). This implies that the model as a whole is statistically acceptable.
Table 2: Model Diagnostic

<table>
<thead>
<tr>
<th>Statistic</th>
<th>RM1</th>
<th>GDP</th>
<th>Inf</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fk=1(3,32)</td>
<td>1.80</td>
<td>22.09**</td>
<td>34.78**</td>
</tr>
<tr>
<td>Fk=2(3,32)</td>
<td>0.30</td>
<td>0.28</td>
<td>6.31**</td>
</tr>
<tr>
<td>Fk=3(3,32)</td>
<td>2.06</td>
<td>0.41</td>
<td>24.03**</td>
</tr>
<tr>
<td>Fk=4(3,32)</td>
<td>5.67**</td>
<td>1.68</td>
<td>14.24**</td>
</tr>
<tr>
<td>Fau(4,30)</td>
<td>0.93</td>
<td>2.22</td>
<td>0.30</td>
</tr>
<tr>
<td>Farch(4,26)</td>
<td>1.45</td>
<td>0.46</td>
<td>0.75</td>
</tr>
<tr>
<td>Fhet(26,7)</td>
<td>0.39</td>
<td>0.35</td>
<td>0.13</td>
</tr>
<tr>
<td>Normality</td>
<td>0.26</td>
<td>24.75**</td>
<td>1.57</td>
</tr>
</tbody>
</table>

Multivariate tests:
- Fau(36, 59)= 1.28
- Fhet(156, 19)= 0.12
- $\chi^2_{n}(6)= 25.69**$
- Fun(39,95)= 144.88**

**Notes:**
1. * Rejects null hypothesis at 95% significance level, ** rejects null hypothesis at 99% significance level.
2. Fau stands for error autocorrelation test, Farch for autoregressive conditional heteroscedasticity, Fhet for White’s functional form/heteroscedasticity test and $\chi^2_{n}$ for White’s normality test.

7.2. Testing for cointegration

Testing for cointegration by the Johansen method requires testing for the reduced rank or for the number of cointegration vectors. In fact the rank of the matrix $\Pi$ can be determined by testing whether or not its eigenvalues ($\lambda$) are statistically different from zero. There are two test statistics to be used for that purpose: the $\lambda$ trace statistic and the maximal eigenvalues statistic. The eigenvalues obtained from the VAR models, the test statistics and critical values are presented in Table 3. The table also shows Reimers’ adjusted test statistics for small sample bias. Reimers (1992), by using the Monte Carlo studies, suggests taking account of the number of parameters to be estimated in the model, and making an adjustment for the degrees of freedom by replacing $T$ in (15) and (16) and the next formula by $T-nk$, where $T$ is the number of observations, $n$ is the number of variables, and $k$ is the number of lags.
\[
\lambda_{\text{trace}} = -T \sum_{i=r+1}^{n} \ln (1 - \hat{\lambda}_i) \tag{14}
\]

\[
\lambda_{\max} (r, r + 1) = -T \ln (1 - \hat{\lambda}_{r+1}) \tag{15}
\]

Where \(\hat{\lambda}_i\) are estimated eigenvalues and \(T\) equals the number of employed observations.

Table 3 contains eight columns. From the left, the first column represents the rank number. The second shows the values of characteristic roots and eigenvalues corresponding to three combinations of the underlying variables, which are ordered from the greatest value to the smallest. The third column is \(\lambda_{\max}\) which has already been described. The next column is the \(\lambda_{\text{adjusted}}\) (Reimers 1992) value and the fifth column is the critical value for the \(\lambda_{\max}\) with 95% level of confidence. The last three columns correspond to the third, fourth and fifth columns, as applied to \(\lambda_{\text{trace}}\). The first line of Table 3 is to test the null hypothesis of no cointegration with the \(\lambda_{\max}\) statistic, i.e.

\(H_0 : r = 0\) versus \(H_A : r = 1\).

Since for \(r = 0\), the \(\lambda_{\max}\) statistic is 76.45, which is higher than the critical value of 25.5, we reject the null of no cointegration. Since the adjusted value of 60.63 is higher than its critical value, it can support that result as in \(\lambda_{\max}\). Consequently, the null hypothesis (\(H_0 : r = 0\)) can be rejected at the 95% level of confidence for both \(\lambda_{\max}\) and \(\lambda_{\max}\) adjusted. Then we proceed to test.

\(H_0 : r = 1\) versus \(H_A : r = 2\).

The \(\lambda_{\max}\) statistic is 21.18, which is higher than the critical value of 19, so we can reject the null hypothesis that there is a single cointegration vector. The adjusted value of 16.8 being lower than its critical value, it cannot support that result as in \(\lambda_{\max}\).

Now, using the \(\lambda_{\text{trace}}\), we test \(H_0 : r = 0\) versus \(H_A : r \geq 1\).

Since the \(\lambda_{\text{trace}} = 106.3\) is higher than its critical value (42.4), we reject the null hypothesis of no cointegration vector. Proceeding to test \(H_0 : r = 1\) versus \(H_A : r \geq 2\) indicates that we
can reject the null hypothesis of one cointegration vector at 99% significance level since $\lambda_{\text{trace}} = 29.89 > 25.3$.

Reimers’ adjusted tests confirms the first result which is that the null hypothesis cannot be rejected when $r = 0$, since $84.34 > 42.4$ (the critical value for $\lambda_{\text{trace}}$).

For the next test, i.e $H_0: r = 1$ versus $H_A: r \geq 2$, since the calculated value, $23.71$ is less than its critical value ($25.3$), we cannot reject the null hypothesis.

To sum up, the different tests for the reduced rank produced different results. To be precise, the $\lambda_{\text{max}}$ and the $\lambda_{\text{trace}}$ indicate the presence of two cointegration vectors, while the adjusted $\lambda_{\text{trace}}$ and also the adjusted $\lambda_{\text{max}}$ suggest the presence of one cointegrating vector. Such a contradiction in the tests for cointegration is not unusual. However, Reimers (1992) in the Monte Carlo study points out that in small samples the Johansen procedure over rejects the null hypothesis and these contradictory results emerge. He states that this problem can be remedied by a modification proposed by Reisel and Ahn (1988). Their suggestion of using $T - nk$ rather than $T$ adjusts the test statistic consistent with small sample size. However, if 63 observations are not a sufficient sample size, the appropriate diagnostic test for denoting the rank number of the $\Pi$ matrix is the adjusted $\lambda_{\text{trace}}$.

The result is that there is only one cointegration relationship between RM1, GDP, and inflation.

### Table 3: Test of cointegration rank on the variables

<table>
<thead>
<tr>
<th>Ho: r</th>
<th>$\lambda_l$</th>
<th>$\lambda_{\text{max}}$</th>
<th>Adjusted 95%</th>
<th>$\lambda_{\text{max}}$</th>
<th>Adjusted 95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r=0$</td>
<td>0.73</td>
<td>76.45**</td>
<td>60.63**</td>
<td>25.5</td>
<td>106.3**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r=1$</td>
<td>0.30</td>
<td>21.18*</td>
<td>16.08</td>
<td>19</td>
<td>29.89*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r=2$</td>
<td>0.13</td>
<td>8.70</td>
<td>6.90</td>
<td>12.3</td>
<td>8.71</td>
</tr>
</tbody>
</table>

*Rejects null hypothesis at 5% significance level, ** rejects null hypothesis at 1% significance level.

**7. 3. $\beta$ Matrix and testing for cointegration coefficients**

The associated eigenvectors are represented in the rows of Table 4. This table confirms long run coefficients after normalising,

---

6 This term means that dividing all coefficients by dependent variable coefficient.
and contains cointegration vectors (i.e. long-run relationship coefficients). The first row of the table shows the inverse of the signs of variables. The GDP coefficient is positive, because when people obtain more money they tend to hold more real money for spending. This is consistent with all of the conventional theories. The inflation coefficient is negative meaning that in inflationary conditions, people do not wish to hold money, because the value of the domestic currency is declining. Since GDP and inflation are not modeled in the system, they do not need to be interpreted.

\[ RM1 = -0.043t + 0.64GDP - 0.31\text{inf} \] (16)

**Table 4: Normalised Characteristic Vectors (\( \beta \))**

<table>
<thead>
<tr>
<th>RM1</th>
<th>GDP</th>
<th>Inflation</th>
<th>Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>-0.64</td>
<td>0.31</td>
<td>0.04</td>
</tr>
<tr>
<td>-7.97</td>
<td>1.00</td>
<td>0.54</td>
<td>-0.37</td>
</tr>
<tr>
<td>-44.28</td>
<td>188</td>
<td>1.00</td>
<td>-3.39</td>
</tr>
</tbody>
</table>

### 7.4. Comparison and evaluation of the results

We have tried to model narrow money stock through both autoregressive distributed lag (ARDL) and VAR. The ARDL is more appropriate for single equations while VAR is more appropriate for a system. Since GDP and inflation have not been modeled in the system, the estimation of the model by the ARDL and Johansen techniques produce strikingly similar results both in the long-run and in the short-run, indicating its robustness to the estimation techniques. It should be acknowledged here that the ARDL and Johansen method use very different techniques in estimation. The former employs ordinary least squares, while the latter uses the maximum likelihood estimation method. Thus, one-to-one comparison of the magnitudes of the coefficients may not be appropriate and requires some caution. Re writing the long run equilibrium relationships, the ARDL estimates:

\[ ^\wedge RM1 = -0.04 t + 0.38 GDP - 0.19 \text{Inf} \] (17)

While Johansen estimates:

We should suppose that the world inflation is constant.
\( \hat{RM1} = -0.04 t + 0.64 \text{GDP} - 0.31 \text{Inf} \)  

(18)

First of all, both equations have a significant constant term. All coefficients have reasonable signs as expected by the underlying theory. Hence both procedures led to similar results. The maximum number of lags in both procedures is four, and the magnitude of coefficients is very close to each other.

8. Concluding remarks
The first part of this paper discussed the concept of cointegration and its testing by the ARDL procedure. Following the convention, this procedure was then applied to Iranian data in order to investigate the existence of a relationship between \(RM1\) and its fundamental determinants. The estimation results indicated that real narrow money, GDP and inflation form a stable long run relationship, and that all economic variables have sensible signs and magnitudes, as expected by theory. Then we moved to the Johansen procedure. First we tested the number of lags and concluded that it is four. We then specified deterministic components and we concluded that the fourth model of Harris, 1995 is the appropriate model. Then we tested a cointegration test. According to adjusted \(\lambda_{trace}\) there is only one cointegration vector for the \(RM1\) model. Then we derived the long run coefficients for \(RM1\) models and concluded the same results as those of the ARDL test.
Reference: